Retrospective Hotelling's T^2 Control Chart for Automotive Stamped Parts: a Case Study

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Abstract

Traditional univariate control charts designed to monitor single variable quality characteristic have been successfully used in manufacturing processes. However, in manufacturing environment, variable settings are essentially multivariate that univariate control charts are not suitable for monitoring purposes. The Hotelling's T^2 multivariate control chart is a powerful statistical tool for modeling multivariate production systems and can shed light on how variables are interrelated to facilitate better understanding of process variations. This study deploys a multivariate control-charting scheme to monitor the quality of a manufactured part in a Malaysian-based automotive parts manufacturing company, as a case study. Three major steps in the Hotelling's T^2 retrospective operation are outlier deletion, variable selection and parameter estimation are methodically described in this paper. When applied to new sample observations of selected quality variables, the T^2 control chart reveals an 'out-of-control' condition, thus confirming the need for quality enhancement in the locally produced stamped part.

Keywords: hotelling's T^2; control chart; automotive stamped;

1. INTRODUCTION.

Traditional Malaysian quality management practices are being challenged by the new quality assurance paradigm of statistical process control (SPC). SPC uses statistical techniques to monitor and control product quality, where control charts are deployed as test tools for monitoring the production process. Conventional univariate control charts designed to monitor a single variable quality characteristic have been successfully used in manufacturing processes. However, in manufacturing environment, the variable settings are essentially multivariate in nature and, under these circumstances, the univariate control charts are not suitable choices for monitoring purposes. Hotelling's T^2 control chart developed by Harold Hotelling in 1947, is a powerful statistical method to improve the quality of products and industrial processes by understanding and monitoring their multi-dimension and multifaceted nature. The charting technique takes into consideration the correlation between the variables in constructing the parameters of multivariate control chart which are mean vector and variance-covariance matrix [1]. While theoretical research in multivariate control charts has been diversified and reported to be at its highest level with the increased measurement and computing ability; there is a dearth of studies on the application of multivariate T^2 control charting technique in the field of automotive stamped parts manufacturing to monitor quality. The majority of past empirical studies have been devoted to the univariate control charting techniques [2-4] and only a few studies employ the Hotelling T^2 multivariate control chart with the technique of principal component analysis [5, 6]. This study deploys the multivariate control charting scheme to monitor the quality of a manufactured part in a Malaysian-based automotive body and parts manufacturing company. Among the reasons for choosing the techniques are, firstly, population parameters of the automotive stamped data are unknown and secondly, the mean shifts in the parts geometrical dimension are not insignificant [7-10]. This fact has been revealed in a number of previous empirical studies where it is reported that automotive stamping process bound to produce large mean shifts in its stamped parts [2] .Apparently, a number of past studies also claim that the procedure of control charting which is suitable for automotive panel stamping process is control chart for individual observations [2].

The next section describes the background of this case study undertaken in a local automotive stamped parts manufacturing company followed by the methodology of the retrospective Hotelling's T^2 control charting scheme for an automotive stamped part manufactured. Since multivariate control charting technique is totally new to the company, the parameters are unknown and have to be estimated, the distinctive features of Retrospective phase is crucial and must be made clearly. This study attempts to elaborate the execution of standard Hotelling's T^2 control charting application to the automotive stamped part data and contributes towards the practice of statistical process control in manufacturing processes. Section 2 clearly describes the process. Section 3 presents results and findings from charting the T^2 control chart followed by some discussions before the concluding remarks and future research direction in the final section.

2. MATERIALS AND METHOD.

The study is conducted in a major Malaysian-based automotive part manufacturing company. The company manufactures various automotive parts such as car roof, bonnet, fender, door and body panels. This company also specializes in the design, engineering and manufacturing of dies and moulds used to produce the automotive stamped panels or parts. The automotive part selected for this study is Reinforced Rear Floor Side Member. It is an inner part reinforcing the rear or back floor located on right side of a national car model. A sample of 140 parts was selected to be analyzed. The sample design is based on individual observations or rational sub-grouping of size one where 35 panels are selected from five different production runs operated on different days [11]. In this study, the quality features measured from the automotive stamped part are surface and trim. A total of 10 points of geometrical dimensions are measured for surface and trim. The measuring work is only performed manually by using hard measuring fixtures (or jig) and measuring equipment. This is due to limited Coordinated Measuring Machine (CMM) facility in within the company. All the measured quality data comprises the historical data set (HDS) for the study and assumed to be continuous variables. All measurements represent the deviations of geometrical dimensions from their respective nominal values in unit millimeter (mm). Table 1 displays the notations of surface and trim variables and the critical-to-quality (CTQs) for both quality features selected for this study.

Table 1: Notations and specification limits of the quality variables

Quality Variable	Notations	Specification value (mm)
Surface (**SP***i*)	gap between panel's flange and jig's surface	3.0 ±0.5
Trim (**TP***i*)	measurement lengthwise from the panel's flange end to the trim line on the jig	2.3 ±0.5

Note: *i* is the number of respective measure point on the panel

2.1 The Retrospective Hotelling's T^2 Control Chart Technique

Figure 1 shows two different stages under the retrospective phase of Hotelling's T^2 charting operation [6, 12]. As multivariate control charting technique is totally new to the company under study, the parameters are unknown and have to be estimated. Additionally, a clear distinction between the two stages of Retrospective phase is crucial for this study [13]. Stage I includes the development of reference sample obtained from the historical data set (HDS). Reference sample is a sample believed to form a process which is in 'statistical control' from which parameters would be estimated from. The estimated parameters are, then, utilized in Stage II to evaluate the control charting scheme when a new set of data is applied. If there is presence of any mean shifts in the quality variables of the new dataset, further analysis is made. Under the stage of developing a reference sample, the Hotelling's T^2 technique is utilized to identify any outliers. The T^2 technique provides a simple and helpful procedure in locating individual outlying observations by identifying mean shifts and distributional deviations from 'in-control' sample distributions.

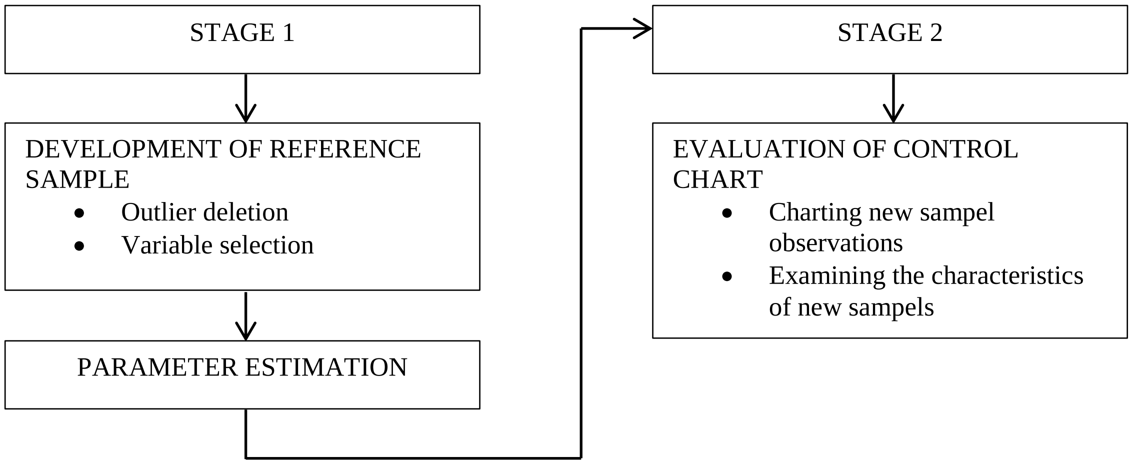


Figure 1: Stages in Retrospective Hotelling's T^2 control charting scheme

The value of α or the probability of Type I error applied would determine the size of control region where observations are statistically much away from the mean vector i.e. the outliers [14]. During the outlier purging process, all observations whose T^2 values are found to be greater than the upper control limit (UCL) are regarded as outliers hence discarded from the data set. Those observation less than or equal to the UCL are deemed as statistically in control thus remain in the data set. New estimates of mean vector and covariance matrix are computed from the remaining set of data for another outlier purging process. This iterative process continues until a homogeneous or 'in-control' set of observations is obtained. Outlier deletion is a crucial process that failure to discard the outliers will deteriorate the actual process of parameter estimation for the control chart scheme. More importantly, the estimated parameters are to be utilized for the actual monitoring purposes during the prospective phase. The next step is to select the 'most appropriate' quality variables to establish the reference sample. To do this, from a large number of variables is to be reduced by applying the Principal Component Analysis (PCA) approach of factor analysis technique. Here, PCA technique aims to describe quality in terms of a smaller number of factor components based on total variation in the measurement of the quality features. With reduced number of quality variables, the established reference sample is now ready for a parsimonious analysis. A set of unbiased mean vector and covariance matrix estimated from the reference sample are the parameters employed in computing T^2 statistics and upper control limit (UCL) for Stage I retrospective T^2 control chart. The other constituents which determine the control chart scheme are the size of reference sample (N) and the number of quality variables (p). Based on the T^2 statistics and UCL, the second stage of retrospective T^2 charting scheme is subsequently applied to a new sample observations to evaluate the effectiveness of the control chart scheme by detecting any departures from the parameters estimated from the established reference sample [15].

2.2 The Hotelling's T^2 Statistics

The Hotelling's T^2 statistics is given as $T^2 = (\boldsymbol{X} - \boldsymbol{\mu})' \Sigma^{-1} (\boldsymbol{X} - \boldsymbol{\mu})$ [16], where $\boldsymbol{X_i}$ is the p-component vector of observation i (i = 1, 2, n). $\boldsymbol{X}_i' = \lfloor X_{i1}, X_{i2}, ..., X_{ip} \rfloor$ comprising p quality characteristics monitored simultaneously, and the nominal vector of means of the $\boldsymbol{X}$'s is given as $\boldsymbol{\mu}' = \lfloor \mu_1, \mu_2, ..., \mu_p \rfloor$. The variances and covariances of random variables X can be shown in a p x p covariance matrix**.** The T^2 statistics is the generalized form of the squared distance from $\boldsymbol{X_i}$ to $\boldsymbol{\mu}$ assuming the distribution of p quality variables are p-variate normal, N_p ($\boldsymbol{\mu}$, $\boldsymbol{\Sigma}$). If $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are both unknown, the estimator of the mean vector $\overline{\boldsymbol{X}}_m$ and the covariance matrix $\boldsymbol{S_m}$ are obtained from m individual sample observations (with n = 1) are applied to compute the T_i^2 shown as $T_i^2 = \left(X_i - \overline{\boldsymbol{X}}_m\right)' \boldsymbol{S}_m^{-1} \left(X_i - \overline{\boldsymbol{X}}_m\right)$ where i = 1, 2, …, m and m is the number of sample observations. For the retrospective control charting operation, the upper control limit (UCL) of the T_i^2 statistics in above equation is based on a beta distribution given as $UCL = \frac{(m-1)^2}{m} B_{\alpha, p/2, (m-p-1)/2}$ [1] where $B_{\alpha, p/2, (m-p-1)/2}$ is the upper α percentage point of a beta distribution with parameters $p/2$ and $(m-p-1)/2$. In the second stage the mean vector $\overline{\boldsymbol{X}}_m$ and covariance matrix $\boldsymbol{S_m}$, are utilized to calculate the T_f^2 statistics of the prospective phase T^2 chart. Here, the T_f^2 statistics is defined as $T_f^2 = \left(X_f - \overline{\boldsymbol{X}}_m\right)' \boldsymbol{S}_m^{-1} \left(X_f - \overline{\boldsymbol{X}}_m\right)$, where f is the future individual sample observation. The upper control limit based on F-distribution is computed by the formula $UCL = \frac{p(m+1)(m-1)}{m(m-p)} F_{\alpha, p, m-p}$. In the control charting technique for individual observations, where n = 1, the univariate control charts for individuals (X) and for moving ranges (MR) are often used for the study of individual observations (Champ et al. 2005). The Hotelling's T^2 statistics approach to multivariate quality control chart for individual observations therefore measures the significance shifts from the out-of-control mean vector, $\boldsymbol{\mu}_s$ to the nominal mean vector $\boldsymbol{\mu}$, by testing the hypothesis: $H_0; \boldsymbol{\mu}_i = \boldsymbol{\mu}$ vs. $H_0; \boldsymbol{\mu}_i \neq \boldsymbol{\mu}$

3. RESULTS AND DISCUSSION.

3.1 Variance Analyses

The analysis of variance (ANOVA) and variance component analysis are two variation analysis applied to the HDS. The HDS comprises of 9 surface and 8 trim quality measurements for each of the 140 stamped parts. ANOVA is applied to 10 points of surface and trim on the stamped parts. These points are the critical-to-quality (CTQs) of surface and trim to examine if there is significant variation in the quality variables across different production runs. Table 2 presents the summary results of ANOVA for surface and trim. The table shows mean differences in all surface variables except SP3. The analysis also indicates mean differences in trim variables. In addition, Table 2 significantly points out the existence of large part-to-part variation in the stamped part quality variables. To further explicate this, the variance component of each data set is analyzed. Component of variance is a measure the extent of how much variance is attributable between different factor level and variability within factor levels i.e. random error. The analysis reveals that large percentage of the variation in surface and trim data is attributable to the random error or error within the individual observation. For both surface and trim, within-individual measurement variation account for more than 90 percent of the total variation and only a minimal of 7-9 percent is assigned to between-run variation. Table 3 exhibits the summary of the variance component analysis for the two variables.

Table 2: Summary of ANOVA results of the surface and trim data

		p-value		p-value			p-value		p-value
Between Run	SP1	0.000*	TP1	0.000*	Between Subgroups	SP1	<0.000*	TP1	<0.000*
	SP3	0.044*	TP3	<0.00*		SP3	0.120	TP3	<0.000*
	SP9	0.143	TP9	0.004*		SP9	0.040*	TP9	<0.000*
	SP11	-@	TP11	0.156		SP11	0.003*	TP11	0.0004*
	SP12	0.0423	TP17	0.036*		SP12	0.0191*	TP17	<0.000*
	SP26	0.000*	TP26	0.006*		SP26	<0.000*	TP26	<0.000
	SP28	0.019*	TP30	0.144		SP28	0.001*	TP30	<0.001*
	SP30	-@	TP32	0.002*		SP30	0.000*	TP32	<0.000*
	SP32	0.034*				SP32	0.000*		

Note 1: * $p < 0.005$
Note 2: @ Main effect ANOVA was not performed due to negative variance component

Table 3: Variance component of surface and trim (% of total variation)

	Bet. Run	Bet. Subsample	Within		Bet. Run	Bet. Subsample	Within
SP1	12.7	29.8	7.5	TP1	10.6	36.4	3.0
SP3	4.4	3.7	91.9	TP3	13.7	27.6	8.7
SP9	1.1	10.2	88.6	TP9	1.4	68.2	0.3
SP11	2.3	15.6	82.2	TP11	2.9	19.2	7.9
SP12	3.6	10.6	85.7	TP17	5.0	54.7	0.3
SP26	17.4	27.5	55.0	TP26	5.4	26.8	7.8
SP28	4.4	17.7	77.8	TP30	2.9	16.9	0.3
SP30	2.9	16.9	80.3	TP32	6.9	37.2	6.0
SP32	2.6	23.7	73.7				
Average%	5.71	17.30	76.97	Average%	6.10	35.88	58.04

Findings on the large part-to-part or within variation in quality variables of automotive stamped parts support the earlier studies on automotive stamping variation which reported large mean deviations in stamped parts quality variables [4]. Additionally, this study supports several previous studies claiming that the appropriate control charting procedures for automotive stamped parts manufacturing is control charts for individual observations [2]. The findings justify the choice of retrospective T^2 control chart for individual observations design scheme for this study as most suitable.

3.2 Retrospective Control Chart: Stage 1

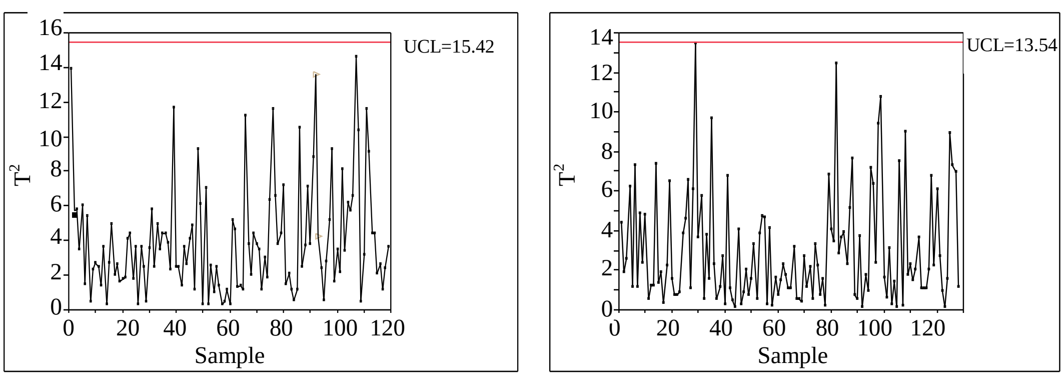
Stage I of the retrospective phase begins with the process of outlier deletion by charting the T^2 control chart to identify the observations which exceeds the upper control limit computed at $\alpha = 0.001$ [12]. At retrospective phase, a small value of α is recommended to reduce the chance of excluding too many observations at the early stage. The purging process ends with a total of 18 outliers of surface variable being detected and, hence, purged out. The PCA also facilitates the selection of variables by identifying the factor components contribute most variation. The rule of thumb is to select those factors with eigenvalue greater than unity [14]. The output of PCA on correlation indicates three factor components accountable for variation in surface and two factor components for trim data set. However, only two factor components are chosen in this study because there is only a small number of quality variable available, PCA selects the variables by analyzing factor loadings of more than 0.60 [17] through varimax rotation with Kaiser normalization (KMO>0.5 and Bartlett's Test of Sphericity p-value = 0.000). Table 4 presents two quality variables from each factor which are selected ; SP9 and SP11 for Factor 1 and SP26 and SP28 are selected for Factor 2. For trim variables TP9 and TP17 are selected under Factor 1 and only TP32 for Factor 2.

Before estimating for the parameters i.e. mean vector and covariance matrix, it is essential to further check if the new reference sample comprising the selected variables is 'in-control' by examining any remaining outliers. For this purpose, the Hotelling's T^2 control chart at $\alpha = 0.0027$ is plotted. At this stage the α is marginally higher than the one applied for the outlier deletion process[18, 19]. At this stage, three observations are purged out from the surface reference sample leaving 119 observations for the subsequent process of parameter estimation. To check for the state of statistical control of trim reference sample, series of Hotelling's T^2 control is applied at $\alpha = 0.0027$ finally produce the remaining 128 observations. Figure 3 illustrates the Hotelling's T^2 chart for the established surface and trim reference sample. The resulting established reference sample for the automotive stamped parts quality variables comprised of 119 and 128 observations for the surface and trim variable, respectively. Table 5 presents the descriptive statistics and the estimated pool covariance matrix and mean vector of the reference sample for both variables.

Table 4: Variable selection based on the Principal Components of the Quality Variables

Principal Components : on Correlation — Factor Rotation: Varimax — Prior Communality Estimates : 1

Number	Eigenvalue	Percent	Cum Percent	Rotated Factor Pattern	Factor 1	Factor 2
SURFACE						
1	2.2525	25.027	25.027	SP1	0.007381	-0.247214
2	1.9890	22.100	47.128	SP3	-0.459225	-0.176074
3	1.2169	13.521	60.649	SP9	0.6981766	0.1686446
4	0.9248	10.275	70.924	SP11	0.7693708	0.0904652
5	0.7399	8.222	79.146	SP12	0.6509998	-0.185914
6	0.6649	7.388	86.534	SP26	0.0955908	0.9053843
7	0.4798	5.332	91.865	SP28	-0.258101	0.7601266
8	0.4589	5.098	96.964	SP30	0.2719365	0.7091209
9	0.2733	3.036	100.00	SP32	0.5382762	-0.186854
TRIM						
Number	Eigenvalue	Percent	Cum Percent	Rotated Factor Pattern	Factor 1	Factor 2
1	3.3208	41.5510	41.510	TP1	-0.453298	-0.70957
2	1.1994	14.992	56.502	TP3	-0.762282	-0.0763
3	0.9208	11.510	68.012	TP9	0.838808	-0.12321
4	0.7416	9.270	77.282	TP11	0.555513	-0.10064
5	0.5757	7.196	84.478	TP17	0.821877	-0.1215
6	0.5425	6.781	91.259	TP26	-0.53957	0.38781
7	0.4214	5.268	96.527	TP30	-0.52359	0.440357
8	0.2779	3.473	100.00	TP32	-0.35037	0.681317



Surface

Trim

Figure 3: The Hotelling's T^2 chart of the 'in-control' sample of surface and trim data (Note: UCL is calculated based on α =0.0027)

Table 5: Descriptive Statistics and Covariance Matrix of Surface and Trim Reference Sample

Quality Variable	SP9	SP11	SP26	SP28		TP9	TP17	TP32
Sample Observations	119	119	119	119		128	128	128
Sample Size	1	1	1	1		1	1	1
Mean	2.60000	2. 7563	3.17899	3.35546		1.65859	1.86484	2.13125
Std Deviation	0.10814	0.09294	0.20538	0.12261		0.15396	0.18931	0.23468
Covariance								
SP9	0.0117	0.0049	0.0037	-0.0016	TP9	0.0237	0.0023	-0.0020
SP11	0.0049	0.0086	0.0024	-0.0020	TP17	0.0023	0.0358	0.0027
SP26	0.0037	0.0024	0.0422	0.0160	TP32	-0.0020	0.0027	0.0551
SP28	-0.0016	-0.0020	0.0160	0.0150				

3.3 Retrospective Control Chart: Stage 2

To evaluate the effectiveness of the T^2 control charting scheme, a sample set comprising 35 stamped parts are collected from a separate production run. The T^2 statistics of the new sample observations are computed based on mean vector and covariance parameters estimated from the 'in-control' reference sample during Stage 1. The T^2 statistics are plotted with UCL calculated at α = 0.0027. During Stage 2 operation, the distribution of T^2 statistics follows an F-distribution which likely describe the T^2 statistics based on the underlying statistical assumption [19]. The Hotelling's T^2 chart is plotted with the UCL computed as 17.89 (α = 0.0027). The T^2 chart shows that the process stamping of surface variables is not 'statistically in-control'. Four 'out-of-control' signals are identified. The new sample observations of trim variables, apparently, are all below the UCL of 15.28 indicating a stable 'process'. See Figure 4.

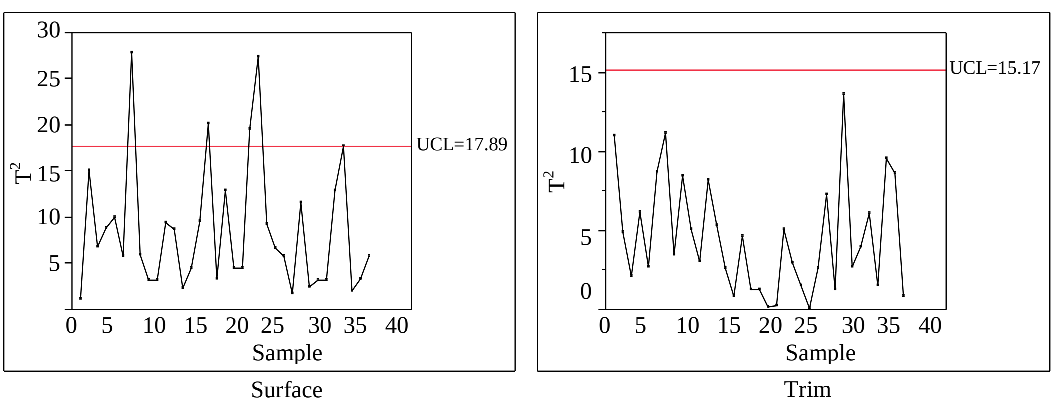


Figure 4: The Hotelling's T^2 chart of the new sample observations of quality data (Note: UCL is calculated based on α = 0.0027)

By applying the Retrospective Hotelling's T^2 control charting scheme to the new set of automotive stamped parts data, an out-of-control condition is revealed by the new sample surface observations. In contrast, the variables of new trim sample observations are all statistically in control. The final step is to check for the causes of out-of-control condition occurred within the new surface sample observations. Two aspects examined are the correlation structure and difference of mean vectors of the new sample. The analysis on surface reference sample indicates positive relationship between variables SP9 and SP11 (r = 0.49 with p = <0.0001) and between variables SP26 and SP28 (r = 0.64 with p = <0.0001). The correlation structures between the same pair variable of the reference sample, nevertheless, are relatively lower. Moreover, the correlations are not significant (SP9 and SP11: r = 0.1066, p = 0.5421; SP26 and SP28: r = 0.0575, p = 0.7429). This indicates that there is presence of assignable causes in the new sample observation which aggravates the general positive trend of SP9 and SP11, as well as SP26 and SP28. The mean vectors of new sample data is examined by employing Tukey-Kramer HSD mean multiple comparison tests for batch means, results of the test indicate that the mean vector of production run 5 is significantly different from mean vectors of batch 1, 2, and 4, but not significantly different from mean vector of the batch of production run 3. The four batches belonging to the reference sample are denoted by number 1 to 4, while the sample of new observations is denoted by number 5. Table 6 presents results of the multiple comparison tests.

Table 6: Tukey-Kramer HSD Multiple Comparisons of pairs for the Surface mean variables of five production runs

Batch	Batch	Difference	Std Err Diff	Lower CL	Upper CL	p-value
5	2	0.322684	0.040461	0.21094	0.434424	0.0000*
5	1	0.309048	0.041489	0.19447	0.423627	0.0000*
5	4	0.277566	0.042712	0.15961	0.395523	<0.0000*
5	3	0.079286	0.042279	-0.03747	0.196048	0.3352

In short, the correlation structure of surface for the new set of 35 sample observations does not resemble those of the reference sample and the mean vector of the new samples are significantly different from means of the four batches comprising the reference sample too. In general, these findings reveal that the process characteristic of new sample differs from the characteristic of reference sample. The control chart shows large erratic behavior of the T^2 statistics. Further investigations on the actual production of stamping process where the new observations are sampled unearth the occurrence of several production interruptions. Such condition may seriously affect the quality of the panel surface that could have contributed to the out-of-control condition of the control chart for the new sample data.

This study uncovers two nature characteristics of automotive stamping process could have effect on the performance of the standard Hotelling's T^2 chart. The first characteristic of automotive stamping process is the existence of large mean shifts in the stamped parts quality variables. Secondly, the nature of batch process of the stamping process has an effect on the mean deviations of the quality variables. The two nature of stamping process have been revealed in several empirical studies on automotive stamping process. Previous studies reveal that the activity of die setups occur in batch process has caused mean shifts in the quality variables [2]. It is also reported that there is no simple adjustment that can be made to shift back these mean dimensions. In short, the mean shifts are inherent and due to this, it is likely the mean of every dimension of a panel not at its nominal specifications [4].

4. CONCLUSION

The multivariate control charts such as Hotelling's T^2 control chart could be considered as powerful statistical tool for modeling multivariate production systems in industry; and can shed light on how variables are interrelated to facilitate a better understanding of process variations. This study deploys a multivariate control-charting scheme, Specifically, the retrospective Hotelling's T^2 control chart for individual observations to monitor the quality of a manufactured part in a Malaysian-based automotive body and parts manufacturing company, as a case study. This paper contributes toward the practice of statistical process control in manufacturing processes. The study elaborates on the application of the retrospective T^2 control chart for a set of data on automotive stamped parts manufacturing which includes outlier deletion and variable selection from the historical data set to establish a reference sample. This study implies how the setting up of process components has impacted the output component. The findings also show that the nature of the batch production of stamping process and the inherent characteristics of shifts in mean vector may render the standard Hotelling's T^2 control chart ineffective in monitoring the quality of automotive stamped parts. The future direction of this study is to formulate an adaptive version of T^2 control chart to take into account the inherent nature of the automotive parts production process.

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